The little hierarchy in Universal Extra Dimensions

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ABSTRACT: In the standard model in universal extra dimensions (UED) the mass of the Higgs field is driven to the cutoff of the higher-dimensional theory. This re-introduces a small hierarchy since the compactification scale $1/R$ should not be smaller than the weak scale. In this paper we study possible solutions to this problem by considering five-dimensional theories where the Higgs field potential vanishes at tree level due to a global symmetry. We consider two avenues: a Little Higgs model and a Twin Higgs model. An obstacle for the embedding of these four-dimensional models in five dimensions is that their logarithmic sensitivity to the cutoff will result in linear divergences in the higher dimensional theory. We show that, despite the increased cutoff sensitivity of higher dimensional theories, it is possible to control the Higgs mass in these two scenarios. For the Little Higgs model studied, the phenomenology will be significantly different from the case of the standard model in UED. This is due to the fact that the compactification scale approximately coincides with the scale where the masses of the new states appear. For the case of the Twin Higgs model, the compactification scale may be considerably lower than the scale where the new states appear. If it is as low as allowed by current limits, it would be possible to experimentally observe the standard model Kaluza-Klein states as well as a new heavy quark. On the other hand, if the compactification scale is higher, then the phenomenology at colliders would coincide with the one for the standard model in UED.

KEYWORDS: Beyond Standard Model, Field Theories in Higher Dimensions, Technicolor and Composite Models.

Contents

1. Introduction

Theories with Universal Extra Dimensions (UED) have attracted considerable attention recently [1]. These theories afford the possibility that the compactification scale $1/R$ is not far above the weak scale M_W, since the propagation of all fields in the extra dimensional bulk generates selection rules derived from momentum conservation. More concretely, after compactification, the conservation of Kaluza-Klein number in any given vertex involving Kaluza-Klein (KK) excitations implies that at least two KK modes must be present in an interaction with a zero mode. As a consequence, the first KK excitations must be pair produced at colliders, which lowers the direct search limits with respect to s-channel production. KK number conservation also means that electroweak precision constraints can only be affected by one loop contributions involving KK modes. Although KK-number conservation is broken by the presence of boundary terms, these typically result in rather suppressed KK-number violating couplings, leaving the bounds on $1/R$ still rather low and not much above the weak scale. This fact has sparked several phenomenological studies about signals for UED at colliders, where the standard model (SM) in the bulk has been used as the theory.

However, the SM in UED is not a natural theory, even if $1/R$ is of the order of the weak scale. This is due to the fact that the bulk Higgs field is inevitably affected by quadratic divergences that are only cut off at a scale Λ_{5D} parametrically larger than $1/R$. In order

to see this let us consider a simple model of a scalar field in five dimensions with the action given by

$$S_\Phi = \int d^4x\, dy\, \left\{ (D_M \Phi)^\dagger D^M \Phi - V(\Phi) \right\}, \tag{1.1}$$

with the potential generically written as

$$V(\Phi) = M^2\, \Phi^\dagger \Phi + \lambda_5 (\Phi^\dagger \Phi)^2. \tag{1.2}$$

In eq. (1.1) the covariant derivative is $D_M = \partial_M + i g_5\, A_M$ and the 5D coupling is given by $g_5 = g\sqrt{2\pi R}$ in terms of the 4D coupling g. The theory of (1.1) is non-renormalizable, with a cutoff Λ_{5D} defined as the scale where the couplings become strong. For instance, for the gauge coupling this implies

$$N\, \frac{g_5^2}{\ell_5}\, \Lambda_{5D} \simeq 1, \tag{1.3}$$

where $\ell_5 = 24\pi^3$ is the 5D loop suppression factor and N is the size of the gauge group. Eq. (1.3) defines the cutoff in the usual sense of naïve dimensional analysis (NDA): the scale for which loops are unsuppressed. With this definition, the interval between the compactification scale and the cutoff of the theory is

$$\Lambda_{5D} R \simeq \frac{12\pi^2}{g^2\, N}, \tag{1.4}$$

and defines a maximum value of the Kaluza-Klein (KK) number for which the KK modes are weakly coupled.

The central question regarding the scalar theory of eq. (1.1) is what is the natural value for the mass parameter M. The existence of the potential of eq. (1.2) implies the presence of radiative contributions to both M^2 and λ_5. This results in a contribution to the mass squared schematically given by

$$N\, \frac{g_5^2}{\ell_5} \int \frac{d^5k}{k^2} \simeq \left(\frac{g^2\, N}{36\pi^2} \Lambda_{5D} R \right) \Lambda_{5D}^2. \tag{1.5}$$

The first factor in the last expression in eq. (1.5) is of order one: the loop suppression is canceled at the cutoff Λ. Thus, radiative corrections naturally give a value

$$M \sim \Lambda_{5D}, \tag{1.6}$$

and in the absence of fine adjustments this should be the typical size of the scalar bulk mass. If we consider the KK expansion, the KK modes have masses

$$m_{(n)}^2 = M^2 + \frac{n^2}{R^2}. \tag{1.7}$$

Then, if Φ were the Higgs doublet in one extra dimension, its zero-mode would have a mass naturally at the cutoff $\Lambda_{5D} \gg 1/R$. This fact is not changed by the presence of fermions or electroweak symmetry breaking, and it constitutes what we have called the little hierarchy problem of the SM in universal extra dimensions.

Although not as marked as the hierarchy problem of the SM in four dimensions, this hierarchy between $1/R$ and Λ implies significant fine tuning in order to keep the Higgs zero-mode light. A natural solution to this little hierarchy problem requires a symmetry in the bulk forbidding the potential for the Higgs field. Furthermore, this symmetry must be broken so as to generate a potential leading to a mass of the order of the weak scale for the scalar.

In this paper we explore the possibility of building a model where the Higgs is a (pseudo-) Nambu-Goldstone boson in UED. Several ideas along this line have been implemented in 4D in the last few years. Here we will implement a Little Higgs model [2 – 4] and a Twin Higgs model [5 – 7] in UED.

In Little Higgs models a global symmetry is spontaneously broken giving rise to Nambu-Goldstone Bosons (NGBs). Part of the global symmetry is gauged, so that gauged interactions explicitly break the global symmetry leading to a radiatively generated potential. The Higgs is then a pseudo-NGB. The potential is now logarithmically dependent on the cutoff of the non-linear sigma model. However, when going to a five-dimensional theory, this tame logarithmic divergence turns into a linear divergence. We will show that, in spite of this, it is possible to stabilize the Higgs mass at the weak scale in Little Higgs theories. As an example we will work in the Simplest Little Higgs [3, 4]. The resulting Higgs mass is somewhat heavier than in the 4D case, $m_h \sim 250$ GeV. This is due to the effects of the KK modes of the top. The solutions we find in this case imply that the compactification scale $1/R$ more or less coincides with the scale where the zero-modes of the Little Higgs "partners" should be. This has important consequences for the phenomenology of UED models.

We will also implement the Twin Higgs model of ref. [5] in UED. In the 4D model it is possible to eliminate the cutoff dependence coming from the top sector, by introducing extra fermions. We show this is still the case in the 5D UED theory. Even with this feature, the Higgs mass is found to be somewhat heavier than in the 4D case, in the range $m_h \sim (170 - 250)$ GeV. On the other hand, and unlike in the Little Higgs case, it is possible for the phenomenology to be the virtually the same as for the SM in UED.

In section 2 we will consider a Little Higgs model, whereas in section 3 we study the use of the Twin Higgs model in UED, both implemented in theories with one compact extra dimension. We conclude in section 4.

2. A little higgs model in UED

In this section we study the possibility of controlling the Higgs mass by assuming that it is a pseudo-Nambu-Goldstone boson. The successful attempts in this direction in four dimensional extensions of the SM require, in addition to having a spontaneously broken global symmetry, an extension of the gauge group as well as new fermions. In these scenarios, called Little Higgs models [2], gauge and Yukawa couplings explicitly break the global symmetry giving the Higgs a mass that is smaller than the Nambu-Goldstone boson (NGB) decay constant f by a loop factor. The new particle content is forced by the global

symmetries to cancel the quadratic divergences in the Higgs mass, leaving only milder logarithmic divergences. Several Little Higgs models are available in the literature [2].

Here we will implement the model first proposed in ref. [3, 4] in UED. The model has an enlarged gauge symmetry: $\mathrm{SU}(3)_w \times \mathrm{U}(1)_X$, which is broken to the SM gauge group by the vacuum expectation values (VEVs) of two scalar triplets. The global symmetry under which these fields Φ_1 and Φ_2 transform is $\mathrm{SU}(3)_1 \times \mathrm{SU}(3)_2$. This is spontaneously broken to $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ by the scalar VEVs. Of the 10 NGBs resulting from this symmetry breaking, 5 will be eaten by the massive gauge bosons resulting from the gauge symmetry breaking. The remaining 5 NGBs remain in the spectrum. In the non-linear description the scalar fields can be parametrized as

$$\Phi_1 = e^{i\Pi \frac{f_2}{f_1}} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix} \qquad \Phi_2 = e^{-i\Pi \frac{f_1}{f_2}} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix} , \tag{2.1}$$

with the NGB fields given by

$$\Pi = \frac{1}{f} \left[\begin{pmatrix} \frac{\eta}{2\sqrt{2}} & 0 & 0 \\ 0 & \frac{\eta}{2\sqrt{2}} & 0 \\ 0 & 0 & -\frac{\eta}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} 0 & 0 & h_1 \\ 0 & 0 & h_2 \\ h_1^* & h_2^* & 0 \end{pmatrix} \right] . \tag{2.2}$$

with η a singlet and

$$h \equiv \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} , \tag{2.3}$$

an $\mathrm{SU}(2)_L$ doublet to be identified with the Higgs field. Although the $\mathrm{SU}(3)_w$ generally breaks explicitly the $(\mathrm{SU}(3))^2$ global symmetry, it respects it at tree level. The kinetic terms and potential for Φ_1 and Φ_2 are invariant under both the global and the gauge symmetries. The explicit breaking induced by the gauge interactions must involve a power of the operator $\Phi_1^\dagger \Phi_2$, as shown in [4]. In order for the $\mathrm{SU}(3)_w$ to generate this kind of operator it should go through a loop such as the one shown in figure 1. This diagram radiatively generates the operator $\left(\Phi_1^\dagger \Phi_2\right)^2$. However, its cutoff dependence is only logarithmic. Thus, only logarithmically divergent loop contributions to the Higgs mass are induced by the explicit breaking. The overall scale of these is determined by the vacuum expectation value breaking the global and gauge symmetries, f. The contribution to the Higgs mass is then of order $f/4\pi$, which is of the order of the weak scale, if $f \sim O(1)\,\mathrm{TeV}$. The gauge symmetry is purposely chosen to explicitly break the global symmetry only in such a way so as to generate logarithmically divergent scalar masses. This is at the heart of this as well as other Little Higgs models.

We would like to implement this model in a UED scenario in order to test if it could be used to solve the Little Hierarchy problem. We will compute the Coleman-Weinberg potential for the zero mode Higgs coming not just from the zero mode spectrum of the Little Higgs model, but also from the KK modes. In order to do this, we implement the cutoff of the 5D theory as a maximum KK number in the 4D effective (KK) theory. Our NDA estimate of eq. (1.4) tells us how high the KK number can be before the theory becomes

strongly coupled. For a typical gauge coupling, using (1.4) results in several dozens of KK modes. However, the Little Higgs model in UED defines a physical cutoff corresponding to that of the non-linear sigma model underlying the theory. The non-linear description can only be valid up to

$$\Lambda \simeq 4\pi f \ . \tag{2.4}$$

The relation between the two cutoffs, and therefore the two scales f and $1/R$, must then be determined. We will not consider the case where f is considerably smaller than $1/R$. Since we still need $f \sim 4\pi v$, this case results in values of $1/R$ which would render the extra dimensions irrelevant for TeV scale physics.
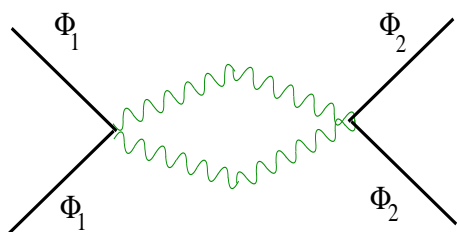

Figure 1: One-loop contribution from gauge bosons to the operator $\left| \Phi_1^\dagger \Phi_2 \right|^2$.

Next, we consider $f > R^{-1}$. This includes the case where both the 5D and the Little Higgs theories have approximately the same cutoff, i.e. $\Lambda_{5D} \simeq \Lambda$. Then we have that

$$f \simeq \frac{3\pi}{g^2 N} \frac{1}{R} \ . \tag{2.5}$$

where $N = N_c = 3$ if we consider the QCD coupling. In this case, the Higgs mass is only effectively regulated above the compactification scale, and therefore can be as large as $1/R$. This would work for low compactification scales, of the order of the weak scale, but is not viable when $1/R$ gets to be close to the TeV scale.

Finally, another possibility is for the two scales to be similar

$$f \simeq \frac{1}{R} \,, \tag{2.6}$$

which would yield a Higgs mass considerably below the compactification scale, allowing us to accommodate experimental limits on KK modes. In this case, the cutoff Λ of the Little Higgs theory from eq. (2.4) will be lower than the maximum energy scale for the KK description to be weakly coupled. Above this cutoff of the non-linear sigma model, there will still be a valid KK mode description, yet the fields expanded in KK modes will correspond now to the ones appearing in the ultraviolet completion (UV) of the Little Higgs model. This is illustrated in figure 2. Above the cutoff for the Little Higgs model, there will be no contributions to the Higgs potential. Then, the effective number of summed KK modes is $n_m = \Lambda R \simeq 4\pi$.

2.1 The mass spectrum of the simplest little Higgs model in UED

We will first review the spectrum of the 4D theory of ref. [4], so we can present its KK expansion later. The matter content is composed by fermions forming $SU(3)_w \times U(1)_X$ representations, in such a way that anomaly cancellation does occur when the three families
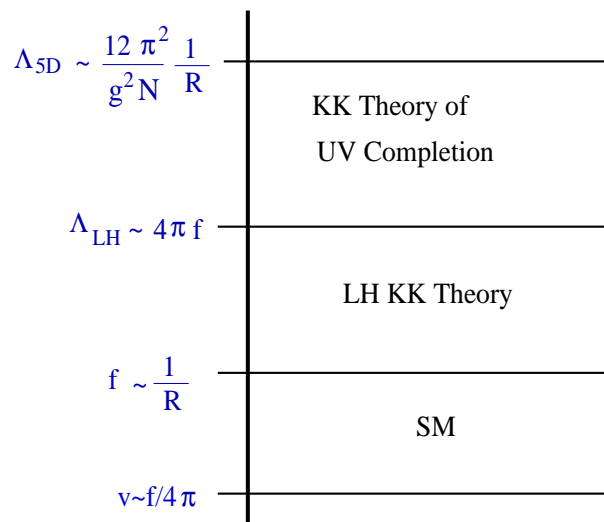
Figure 2: The case with $f \simeq 1/R$. The contributions to the Higgs mass come from scales below the little Higgs cutoff, $\Lambda_{\mathrm{LH}} \sim 4\pi f$. The KK modes above this scale do not contribute to the Higgs potential.

are added together [8] (Model 2 in [4]). For the quarks we have the following set of triplets

$$
\Psi_{Q_1} = \begin{pmatrix} d \\ u \\ D \end{pmatrix}_L \sim (\mathbf{3^*}, 0), \quad \Psi_{Q_2} = \begin{pmatrix} s \\ c \\ S \end{pmatrix}_L \sim (\mathbf{3^*}, 0), \quad \Psi_{Q_3} = \begin{pmatrix} t \\ b \\ T \end{pmatrix}_L \sim (\mathbf{3}, 1/3),
\tag{2.7}
$$

and right-handed singlets

$$
\begin{aligned}
u_R,\ c_R,\ t_R &\sim (\mathbf{1}, 2/3), & d_R,\ s_R,\ b_R &\sim (\mathbf{1}, -1/3), \\
T_R &\sim (\mathbf{1}, 2/3) & D_R,\ S_R &\sim (\mathbf{1}, -1/3).
\end{aligned}
\tag{2.8}
$$

Here, D, S, and T are new quarks which, according to the global symmetry of the little Higgs model, cancel one loop quadratic divergences for the Higgs mass due d, s and t quarks. The scalar triplets in eq. (2.1) transform as $\Phi_1, \Phi_2 \sim (\mathbf{3}, -1/3)$ so that the tree level Yukawa Lagrangian for the quarks is

$$
\begin{aligned}
\mathcal{L}_q = {} & \lambda_{d1} \overline{\Psi}_{Q_1} \Phi_1^* d_R + \lambda_{d2} \overline{\Psi}_{Q_1} \Phi_2^* D_R + \lambda_{s1} \overline{\Psi}_{Q_2} \Phi_1^* s_R + \lambda_{s2} \overline{\Psi}_{Q_2} \Phi_2^* S_R \\
& + \lambda_{t1} \overline{\Psi}_{Q_3} \Phi_1 t_R + \lambda_{t2} \overline{\Psi}_{Q_3} \Phi_2 T_R + h.c.
\end{aligned}
\tag{2.9}
$$

The Yukawa couplings are taken to be diagonal, avoiding cross terms such as $\overline{\Psi}_{Q_3} \Phi_1 T_R$ in order to simplify the analysis. The quarks u, c and b get their masses through higher-dimensional operators. The dominant fermionic contributions to the Higgs mass come from the last two terms in eq. (2.9), corresponding to the top quark and its partner T. Therefore, we disregard all other terms but the last two in eq. (2.9).

Up to order v^4/f^4, the mass squared of the top quark and its partner are (see appendix A for details)

$$m_t^2 = (\lambda_{t1}\lambda_{t2})^2 \frac{f^2 v^2}{M_T^2} \left[1 - \frac{1}{3}\left(\frac{fv}{f_1 f_2}\right)^2 + (\lambda_{t1}\lambda_{t2})^2 \left(\frac{fv}{M_T^2}\right)^2 \right] \tag{2.10}$$

$$m_T^2 = M_T^2 - m_t^2. \tag{2.11}$$

In order to restrict the number of free parameters, we choose λ_{t1} and λ_{t2} so as to minimize M_T for given values of the f_i's, following ref. [4]. The top quark mass then is given by $m_t \approx \lambda_{t1}\lambda_{t2}\frac{fv}{M_T}$, and the top quark Yukawa is

$$\lambda_t = \lambda_{t1}\lambda_{t2}\frac{f}{M_T} \tag{2.12}$$

which fixes λ_{t1} and λ_{t2} to

$$\lambda_{t1} = \sqrt{2}\lambda_t \frac{f_2}{f}, \qquad \lambda_{t2} = \sqrt{2}\lambda_t \frac{f_1}{f}, \tag{2.13}$$

and the heavy top mass to

$$M_T = 2\lambda_t \frac{f_1 f_2}{f}. \tag{2.14}$$

The spectrum's of the zero-mode gauge bosons consists of the neutral gauge bosons Z_1, Z_2 and U^0, with masses (up to order v^4/f^4)

$$M_{Z_1}^2 = m_{Z^0}^2 \left[1 - \frac{1}{3}\frac{v^2 f^2}{f_1^2 f_2^2} + \frac{m_{Z^0}^2}{M_{Z'}^2} \right] \tag{2.15}$$

$$M_{Z_2}^2 = M_{Z'}^2 - M_{Z_1}^2 \tag{2.16}$$

$$M_{U^0}^2 = \frac{1}{2}g^2 f^2, \tag{2.17}$$

where $m_Z^2 = g^2 v^2/2\cos^2\theta_W$ is the tree level squared mass of the SM Z.

Also up to order v^4/f^4 the mass squared of the charged diagonal mass eigenstates $W^\pm$ and $W'^\pm$ are

$$M_W^2 = m_W^2 \left[1 - \frac{1}{3}\frac{v^2 f^2}{f_1^2 f_2^2} + \frac{m_W^2}{M_{\mathrm{Ch}}^2} \right] \tag{2.18}$$

$$M_{W'}^2 = M_{\mathrm{Ch}}^2 - M_W^2 \tag{2.19}$$

where $m_W^2 = g^2 v^2/2$ is the squared mass of the SM W at tree level.

We will now obtain the KK spectrum in the simplest little Higgs model. We consider a theory with one flat extra dimension compactified on a S_1/Z_2 orbifold. In order to obtain the desired zero-mode spectrum, we choose the fermion triplets to satisfy $\Psi(x,-y) = -\gamma_5\Psi(x,y)$. This results in a KK expansion of the form

$$\Psi(x,y) = \frac{1}{\sqrt{2\pi R}}\psi_{0L}(x) + \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\left[\psi_{nL}(x)\cos(\frac{ny}{R}) + \hat{\psi}_{nR}(x)\sin(\frac{ny}{R}) \right]. \tag{2.20}$$

For the singlets, we have $u(x, -y) = \gamma_5 u(x, y)$, resulting in

$$\mathcal{U}(x,y) = \frac{1}{\sqrt{2\pi R}} u_{0R}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[u_{\mathrm{nR}}(x) \cos(\frac{ny}{R}) - \hat{u}_{\mathrm{nL}}(x) \sin(\frac{ny}{R}) \right]. \quad (2.21)$$

The fermion kinetic terms result in the following 4D KK theory:

$$\mathcal{L}_{4D} = \int_0^{2\pi R} dy \ \overline{\Psi} i \Gamma^M \partial_M \Psi$$

$$= \overline{\psi}_{0L} i \gamma^\mu \partial_\mu \psi_{0L} + \sum_{n=1}^{\infty} \left[\overline{\psi}_{\mathrm{nL}} i \gamma^\mu \partial_\mu \psi_{\mathrm{nL}} + \overline{\hat{\psi}}_{\mathrm{nR}} i \gamma^\mu \partial_\mu \hat{\psi}_{\mathrm{nR}} - \frac{n}{R} \left(\overline{\psi}_{\mathrm{nL}} \hat{\psi}_{\mathrm{nR}} + \overline{\hat{\psi}}_{\mathrm{nR}} \psi_{\mathrm{nL}} \right) \right],$$

where $\Gamma^\mu \equiv \gamma^\mu$, $\Gamma^4 \equiv i\gamma_5$ and $\partial_4 \equiv \partial_y$, will have the 4D Little Higgs model as the zero-mode spectrum. Thus, for example, the mass for the top quark KK mode and its partner have the form

$$m_{n,t}^2 = m_t^2 + \frac{n^2}{R^2}, \quad (2.22)$$

$$m_{n,T}^2 = m_T^2 + \frac{n^2}{R^2}. \quad (2.23)$$

For the gauge bosons, we work in the $A_5 = 0$ gauge. For instance, for the abelian field of the $U(1)_X$ gauge factor $B_M(x,y)$, the 5D kinetic term results in the 4D effective Lagrangian

$$\mathcal{L}_{4D}^{g.b} = -\frac{1}{4} \int_0^{2\pi R} dy \ B^{MN} B_{MN}$$

$$= -\frac{1}{4} B_0^{\mu\nu} B_{0\mu\nu} + \sum_{n=1}^{\infty} \left[-\frac{1}{4} B_n^{\mu\nu} B_{n\mu\nu} + \frac{1}{2} \frac{n^2}{R^2} B_n^\mu B_{n\mu} \right] \quad (2.24)$$

and similarly for the non-abelian gauge fields.

2.2 The Coleman-Weinberg potential

The one-loop Coleman-Weinberg potential for the zero-mode Higgs generated by fields having the mass matrix $\mathbf{M_i}(v, f)$ is generically given by [9]

$$V = \frac{1}{64\pi^2} \sum_i N_i \left[2\Lambda^2 \mathbf{Tr}\left(\mathbf{M_i^\dagger M_i} \right) + \mathbf{Tr} \left\{ (\mathbf{M_i^\dagger M_i})^2 \left(\ln \frac{\mathbf{M_i^\dagger M_i}}{\Lambda^2} - \frac{1}{2} \right) \right\} \right], \quad (2.25)$$

where N_i is the number of degrees of freedom for the case of bosonic fields, minus the number of degrees of freedom for fermionic fields, and we have omitted a constant term proportional to the fourth power in the cutoff. Due to the Little Higgs global symmetry, the quadratically divergent term in (2.25) does not contribute to the Higgs mass. We write the relevant part of the potential generated for the Higgs field, by collecting all contributions produced by the SM fields and and their partners. The contribution of each pair can be written as

$$V_i \approx \frac{N_i}{64\pi^2} \left[2\delta_i \ln \frac{\Lambda^2}{M_i^2} + \frac{\delta_i^2}{M_i^4} \left(\frac{1}{2} + \ln \frac{\delta_i}{M_i^4} \right) \right], \quad (2.26)$$

where we have kept terms in δ_i resulting in quadratic or quartic contribution to the Higgs potential. The complete zero-mode contribution to the Higgs potential will then be $V(h^\dagger h) = \sum_i V_i$. Keeping up to quartic terms we have

$$V(h^\dagger h) = m^2 h^\dagger h + \lambda (h^\dagger h)^2 \, , \tag{2.27}$$

where

$$m^2 = -\frac{3}{16\pi^2} \left[2\lambda_t^2 M_T^2 \ln\left(\frac{\Lambda^2}{M_T^2}\right) - \frac{g^2}{4} M_{Z'}^2 \ln\left(\frac{\Lambda^2}{M_{Z'}^2}\right) - \frac{g^2}{2} M_{\mathrm{Ch}}^2 \ln\left(\frac{\Lambda^2}{M_{\mathrm{Ch}}^2}\right) \right], \tag{2.28}$$

and

$$\lambda = \frac{1}{3} \frac{f^2}{f_1^2 f_2^2} |m^2| - \frac{3}{64\pi^2 v^4} \left[4m_t^4 \left(\frac{1}{2} + \ln\frac{m_t^2}{M_T^2}\right) - m_Z^4 \left(\frac{1}{2} + \ln\frac{m_Z^2}{M_{Z'}^2}\right) \right.$$
$$\left. - 2m_W^4 \left(\frac{1}{2} + \ln\frac{m_W^2}{M_{\mathrm{Ch}}^2}\right) \right] , \tag{2.29}$$

which agrees with ref. [4].

As noted in ref. [4], this potential does not result in electroweak symmetry breaking for an acceptably high scale f. This is the result of the mass m in eq. (2.28) being too large and negative. This problem was dealt with in [4] by adding the tree-level "μ" term

$$V_{\mathrm{soft}} = \mu^2 \Phi_1^\dagger \Phi_2 + H.c$$
$$\approx \mu^2 \left[-2f_1 f_2 + \frac{f^2}{f_1 f_2} h^\dagger h - \frac{1}{12} \frac{f^4}{f_1^3 f_2^3} (h^\dagger h)^2 + \frac{f^2}{2f_1 f_2} \eta^2 + \cdots \right] . \tag{2.30}$$

In addition to lower the Higgs mass, the presence of this term explicitly breaks the global U(1) that was keeping the η massless. With this term present the Higgs mass is now

$$m_H = 2v \sqrt{\lambda - \frac{1}{12} \frac{\mu^2 f^4}{f_1^3 f_2^3}} \, , \tag{2.31}$$

allowing us to have a light Higgs for reasonably high values of f (i.e. $\sim TeV$).

We will now consider the contributions of the KK modes to the Coleman-Weinberg potential. As mentioned in section 1, the 5D and the Little Higgs theories are non-renormalizable, and to each of them corresponds a cutoff. In the case with both cutoffs coinciding, the scale f is significantly higher than R^{-1}. Then the SM KK theory populates the region between R^{-1} and f. Above f, the KK theory now is that for the Little Higgs model. In this case, the Little Higgs theory is not efficient regulating m_h, since the large number of KK modes actually contributing to the potential result in an unacceptably large value of m_h.

If on the other hand, $f \sim R^{-1}$, then the Little Higgs cutoff $\Lambda \sim 4\pi f$ appears before the cutoff of the 5D theory. Above this cutoff, the KK theory corresponds to the UV completion of the Little Higgs model, and therefore there will be no contributions to the Higgs potential. We will then sum the KK contributions to the Higgs potential up to a

certain KK number corresponding to the cutoff of the Little Higgs theory, n_m. Generically, for a given pair, the Coleman-Weinberg potential has the form

$$V_i^{kk} = \frac{N_i}{64\pi^2} \sum_{n=1}^{n_m} \left\{ \delta_i + \left(\frac{n^4}{R^4} + m_{i-}^4 \right) \ln \left(\frac{n^4}{\Lambda^4 R^4} + \frac{n^2 M_i^2}{\Lambda^4 R^2} + \frac{\delta_i}{\Lambda^4} \right) \right.$$
$$+ M_i^2 \left(\frac{2n^2}{R^2} + M_i^2 - 2m_{i-}^2 \right) \ln \left(\frac{n^2}{\Lambda^2 R^2} + \frac{M_i^2}{\Lambda^2} - \frac{m_{i-}^2}{\Lambda^2} \right)$$
$$\left. + 2m_{i-}^2 \frac{n^2}{R^2} \ln \left(\frac{n^2 + m_{i-}^2 R^2}{n^2 + M_i^2 R^2 - m_{i-}^2 R^2} \right) \right\}$$
$$\simeq -2\delta_i \frac{N_i}{64\pi^2} \left[n_m - 2\ln \frac{(\Lambda R)^{n_m}}{n_m!} \right] \cdots \tag{2.32}$$

In the last line we omitted subdominant terms, as well as those that do not depend on v. The appearance of n_m, corresponds to the dependence on the cutoff of the Little Higgs theory: $\Lambda R = n_m$. Then if we keep only the dominant behavior with Λ we obtain

$$V_i^{kk} \approx \frac{N_i}{32\pi^2} \delta_i \left(\Lambda R - \ln(2\pi\Lambda R) \right) . \tag{2.33}$$

From eq. (2.33) we can see that the KK contributions introduce a linear cutoff dependence in the Higgs potential, to be compared with the logarithmic dependence for the zero-mode contributions. This is not surprising. As mentioned in section 2, the radiatively generated potential can be seen as coming from the operator $\left| \Phi_1^\dagger \Phi_2 \right|^2$. In the 4D theory, loop diagrams such as the one in figure 1, generated by gauge boson interactions, give rise to this operator with a logarithmic dependence. However, in the 5D theory we can see by power counting that they will result in a linear dependence on Λ. The relevant interaction in the scalar kinetic term of the 5D theory is

$$(D_\mu \Phi_i(x,y))^\dagger D^\mu \Phi_i(x,y) = \cdots + g_5^2 A_\mu(x,y) A^\mu(x,y) \Phi_i^\dagger(x,y) \Phi_i(x,y), \tag{2.34}$$

where $i = 1,2$ and g_5 is the 5D gauge coupling associated with the gauge field $A_M(x,y)$. The one loop contribution to the Higgs mass resulting from the contribution to $|\Phi_1^\dagger \Phi_2|^2$ will then go like

$$\frac{g_5^4 f^2}{(2\pi R)^2} (2\pi R) \int \frac{d^5 k}{(2\pi)^5} \frac{1}{k^4} \simeq \frac{g^4 f^2}{6\pi^2} (\Lambda R) , \tag{2.35}$$

where we used $g_5 = g\sqrt{2\pi R}$ and g is the corresponding 4D gauge coupling. Thus, a 5D Little Higgs must have linear sensitivity to the cutoff Λ. In what follows we explicitly study how this feature affects the effectiveness of the Little Higgs mechanism in controlling the Higgs mass and having satisfactory electroweak symmetry breaking.

The KK modes of the t and T quarks give the following contributions to the quadratic and quartic terms in the potential

$$m_{T_{KK}}^2 = -\frac{3\lambda_t^2 M_T^2}{8\pi^2} \left\{ 2n_m - \ln 2\pi n_m - \sum_{n=1}^{n_m} \left(\frac{n^2}{M_T^2 R^2} + 1 \right) \ln \left(1 + \frac{M_T^2 R^2}{n^2} \right) \right\}, \tag{2.36}$$

$$\lambda_{T_{KK}} = -\frac{f^2 m_{T_{KK}}^2}{3f_1^2 f_2^2} - \frac{3\lambda_t^4}{16\pi^2} \left\{ 2n_m - \sum_{n=1}^{n_m} \left(\frac{2n^2}{M_T^2 R^2} + 1 \right) \ln \left(1 + \frac{M_T^2 R^2}{n^2} \right) \right\}. \tag{2.37}$$

The contributions from the KK modes of the gauge bosons of the model are

$$
m_{g_{KK}}^2 = \frac{3g^2 M_{Z'}^2}{64\pi^2 c_W} \left\{ 2n_m - \ln 2\pi n_m - \sum_{n=1}^{n_m} \left(\frac{n^2}{M_{Z'}^2 R^2} + 1 \right) \ln \left(1 + \frac{M_{Z'}^2 R^2}{n^2} \right) \right\}
$$
$$
+ \frac{3g^2 M_{Ch}^2}{32\pi^2} \left\{ 2n_m - \ln 2\pi n_m - \sum_{n=1}^{n_m} \left(\frac{n^2}{M_{\mathrm{Ch}}^2 R^2} + 1 \right) \ln \left(1 + \frac{M_{\mathrm{Ch}}^2 R^2}{n^2} \right) \right\}, \quad (2.38)
$$

and

$$
\lambda_{g_{KK}} = -\frac{f^2 m_{g_{KK}}^2}{3 f_1^2 f_2^2} + \frac{3g^4}{256\pi^2 c_W} \left\{ 2n_m - \sum_{n=1}^{n_m} \left(\frac{2n^2}{M_{Z'}^2 R^2} + 1 \right) \ln \left(1 + \frac{M_{Z'}^2 R^2}{n^2} \right) \right\}
$$
$$
+ \frac{3g^4}{32\pi^2} \left\{ 2n_m - \sum_{n=1}^{n_m} \left(\frac{2n^2}{M_{\mathrm{Ch}}^2 R^2} + 1 \right) \ln \left(1 + \frac{M_{\mathrm{Ch}}^2 R^2}{n^2} \right) \right\}. \quad (2.39)
$$

Then the linear cutoff dependence appears in both the mass squared and the quartic coupling. In addition to the contributions above, we will also consider the presence of a tree-level μ term such as the one described in eq. (2.30). In the following section we present our results for various representative values of the parameters and discuss them in detail.

2.3 Results and discussion

Putting together all the contributions to the effective potential for the zero-mode Higgs, we will now consider several possible values for the ratio of VEVs

$$
k \equiv \frac{f_1}{f_2} \quad (2.40)
$$

as well as for the μ term defined in eq. (2.30), that result in electroweak symmetry with $v = 246/\sqrt{2}$ GeV. For each successful case we obtain the value of the scale f and of the Higgs mass m_h. The phenomenological viability of a given solution is determined mainly by asking f to be high enough for the heavy states not to have been observed directly, as well as by the requirements from electroweak precision constraints (EWPC). In table 1, we consider the case $f = R^{-1}$, for $\mu = 0$ as well as two other representative values not too different from the weak scale. We also consider different values of k.

The results show that it is possible to have a solution with a Higgs mass sufficiently light for acceptable values of the scale f. This seems to inevitably require the presence of a μ term, just like in the 4D case studied in ref. [4]. The typical value of the Higgs mass for these solutions is around 250 GeV. For instance, for $\mu = 200$ GeV and $k = 0.25$, we obtain a solution with $m_h \simeq 260$ GeV for $f = 1.5$ TeV. This value of f may be at the edge of what can be accommodated by electroweak precision constraints. In order to raise f, we must consider solutions with larger μ's, as it can be seen in table 1. This increases the fine-tuning of the solution (see discussion below). Solutions with $\mu = 0$ give values of f that are too low and excluded by experiment. It should be noticed that, within the approximation made here neglecting the contributions from the to the Higgs potential from light fermions, the resulting potential is symmetric under the exchange $f_1 \leftrightarrow f_2$. Then, table 1 can be completed with the entries for $k \rightarrow 1/k$, which are identical to the ones shown.

μ	k	f	m_h
500	1	1100	246
-	.5	1500	249
-	.25	3400	246
300	1	700	246
-	.5	950	248
-	.25	2100	244
200	1	500	247
-	.5	700	249
-	.25	1500	245

Table 1: Results for the Higgs mass and the scale f, for various values of $k = f_1/f_2$ and the soft μ parameter defined in the text. Note that the results for $k \to 1/k$ are virtually identical (see text for explanation). Here we used $f = R^{-1}$.

We then conclude that, for the case in which $f \sim R^{-1}$, is at least possible to control the Higgs mass with the simplest little Higgs construction of ref. [4], at the cost of a somewhat heavier Higgs. This is due to the linear cutoff sensitivity of the 5D theory, compared to the 4D theory of [4], which is logarithmic.

Another distinct case to consider is when f is parametrically larger than $1/R$. This is of interest, since we can imagine having $1/R \simeq v$ and f sufficiently above that, still allowed by experimental constraints (direct and indirect). But, as discussed at the beginning of this section, if we take $f > 1/R$, this puts the Little Higgs cutoff Λ not far from the 5D cutoff Λ_{5D}. This means that now there would be a larger number of KK modes contributing to the Higgs potential. For instance, for $f = 4\pi R^{-1}$ the number of KK modes contributing is larger than 100. Thus, the linear divergences make it impossible to find acceptable solutions. We then conclude that in this context is not possible to entertain values of R^{-1} as small as 300 GeV, as allowed by experiment. This seems to be a very generic feature of Little Higgs models in 5D.

The presence of the tree-level μ term balancing radiative contributions raises the possibility of fine-tuning in the minimization of the Higgs potential. In order to check for this, we consider as a measure of fine tuning the sensitivity of the Higgs VEV to variations in μ. For this purpose we compute

$$\frac{v^2}{\mu^2} \left| \frac{\partial v^2}{\partial \mu^2} \right| . \tag{2.41}$$

For the solutions with $\mu = 200$ GeV, this is always smaller than 10, signaling a fine-tuning of better than 10%. For example, for $k = 0.25$ this resuls in a 20% fine-tuning. On the other hand, for the solutions with $\mu = 500$ GeV the fine-tuning is already a few percent, being about 3% for $k = 0.25$ and 6% for $k = 1$. Finally, for $\mu = 300\,\text{GeV}$ and $k = 0.25$ we find the fine-tuning to be around 10%. Thus, it is possible to obtain solutions with high enough values of f (so that they are not experimentally excluded), but with reasonable levels of fine-tuning.

Finally, we comment on electroweak precision constraints. For the case of interest, $f \sim R^{-1}$, the main contributions to oblique corrections are still from the zero-mode new gauge bosons, $W'^{\pm}$, the non-hermitian U^0, the neutral Z_2 and a tree level mixing between eigenstates Z and Z'. For the S and T parameters we have [10]

$$S \approx -\frac{1}{4\pi} \frac{M_W^2}{M_{U^0}^2} \left(5 + \frac{2}{3\cos^2 \theta_W} \right) \tag{2.42}$$

$$T \approx \frac{1}{8\alpha} \left(1 - \tan^2 \theta_W \right)^2 \frac{v^2}{f^2} + \frac{1}{4\pi \cos^2 \theta_W} \frac{M_W^2}{M_{U^0}^2} \left(\frac{3}{2\tan^2 \theta_W} \frac{M_{W'}^2}{M_{U^0}^2} - 1 \right)., \tag{2.43}$$

where the first term in eq. (2.43) is due the tree-level mixing $Z - Z'$. The tree level contribution to the T parameter is dominant over the one loop corrections. For the large values of the Higgs mass shown in table 1, agreement with EWPC is somewhat better due to the positive contribution to T. For instance, for $\mu = 200\,\text{GeV}$, $k = 0.25$ and $f = 1.5\,\text{TeV}$, it is found $S \approx -0.006$ and $T \approx 0.20$, still allowed at the 1σ level for $m_h \simeq 260\,\text{GeV}$ [11]. Loop contributions, which dominate S, are decoupling. Therefore, keeping only the zero-mode contributions gives a good estimate of S.

On the other hand, mixing with the new heavy fermions causes shifts in the couplings of gauge bosons with light fermions. Furthermore, four-fermion interactions are induced by integrating out the heavy gauge bosons and are tested by LEP2 data. A full study of the electroweak precision constraints is outside the scope of this work. There is currently no study even for the 4D version of the model we used here (Model II in [4]). However, a full study of the anomalous model (Model I in [4]) in the 4D case reveals that the bounds on the scale f are stronger than the ones derived from only the oblique corrections [13]. If those bounds were to also apply to the UED version of the non-anomalous model, then the allowed solutions would have values of k further from 1 and larger values of the tree-level parameter μ, all of which result in larger values of the scale f. This would push the model to a somewhat more fine-tuned situation, as we discussed above.

The phenomenology of this Little Higgs model in one UED will differ significantly from the corresponding to just having the SM in the 5D bulk [12]. Since $f \simeq 1/R$, the appearance of the KK modes of the SM fields will be accompanied by the presence of the zero modes of the Little Higgs model, both for gauge bosons and for fermions. Thus, the impact of these new states in phenomenological studies at the LHC cannot be ignored.

3. A twin higgs model in UED

We will now study another mechanism to control the Higgs mass in a UED scenario. It was recently pointed out in that discrete symmetries could be used in addition to global symmetries, in order to forbid quadratic divergences from contributing to the Higgs mass [5–7]. We will consider a Twin Higgs model in one UED to see if this mechanism is effective in controlling the divergences arising in extra dimensional theories.

We begin by quickly reviewing the idea behind Twin Higgs models. A Z_2 discrete symmetry is imposed between the SM fields and a "mirror" or "twin" sector which transforms under a mirror gauge symmetry. There is also a global SU(4) symmetry in the Higgs sector. The breaking of the mirror gauge symmetry down to the SM gauge interactions also breaks SU(4) $\rightarrow$ SU(3). The Z_2 symmetry greatly constrains the form of the contributions to the Higgs potential. The gauge group is now SU(2)$_A \times$SU(2)$_B$ with the SM fields transforming under SU(2)$_A$, and their partners transforming under SU(2)$_B$. The Higgs doublet corresponds to 4 of the 7 massless degrees of freedom resulting from the SU(4) $\rightarrow$ SU(3) spontaneous breaking. The Higgs is in the fundamental of the global SU(4)

$$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix} \tag{3.1}$$

where $H_{A,B}$ are doublets of each subgroup $SU(2)_{A,B}$. The gauge interactions explicitly break the global symmetry and generate a potential of the form

$$\Delta V \propto \Lambda^2 \left(g_A^2 H_A^\dagger H_A + g_B^2 H_B^\dagger H_B \right) \tag{3.2}$$

However, the Z_2 symmetry forces $g_A = g_B$, which guarantees that the quadratically divergent contributions in (3.2) are SU(4) symmetric: $\Delta V \propto g^2 \Lambda^2 H^\dagger H$. Thus, these quadratic divergences do not contribute to the Higgs potential. The sensitivity to the cutoff is only logarithmic in the 4D realization. Also in these models is possible to enlarge the global symmetry in the top sector in a way that renders its contributions to the Higgs potential finite. This is a very interesting possibility since the top constitutes the dominant contribution to the Higgs mass. It is also particularly useful in going to a 5D theory, since the cutoff sensitivity is linear instead of logarithmic, as we have seen in the previous section.

First, we review the relevant field content of the Twin Higgs Model [5] we are going to extend to the 5D bulk. The top quark Yukawa couplings have an approximate SU(6) × SU(4) × U(1) global symmetry, with the $(SU(3)_c \times SU(2) \times U(1))_{A,B}$ subgroups gauged. The content of this sector is in the following chiral fermions: $Q_L = (\mathbf{6}, \bar{\mathbf{4}})$ and $T_R = (\bar{\mathbf{6}}, \mathbf{1})$, where we showed the transformations under SU(6) and SU(4), respectively, and we omit the U(1) charge. Their branchings under $[SU(3) \otimes SU(2)]^2$ are

$$Q_L = (\mathbf{3}, \mathbf{2}; \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}; \mathbf{3}, \mathbf{2}) \oplus (\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{2}; \mathbf{3}, \mathbf{1})$$
$$= \quad q_A \quad \oplus \quad q_B \quad \oplus \quad \tilde{q}_A \quad \oplus \quad \tilde{q}_B \tag{3.3}$$
$$T_R = (\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}; \bar{\mathbf{3}}, \mathbf{1})$$
$$= \quad t_A \quad \oplus \quad t_B \,, \tag{3.4}$$

where once again we omitted the U(1) quantum numbers.

The Yukawa interactions are given by

$$\mathcal{L}_q^{\text{twin}} = y H Q_L T_R + h.c$$
$$= y(H_A t_A q_A + H_A t_B \tilde{q}_B + H_A t_A q_A + H_B t_A \tilde{q}_A + H_B t_B q_B) + h.c \tag{3.5}$$

The exotic quarks $\tilde{q}_A$ and $\tilde{q}_B$ get Z_2 symmetric masses with additional fermions $\tilde{q}_A^c$ and $\tilde{q}_B^c$, which have the opposite quantum numbers:

$$M(\tilde{q}_A^c \tilde{q}_A + \tilde{q}_B^c \tilde{q}_B)\,, \tag{3.6}$$

where the mass parameter M is the sole source of SU(4) breaking.

A non-linear realization of the Twin Higgs, has the Higgs in the broken generators corresponding to the seven Nambu-Goldstone Bosons (NGBs) of SU(4)/SU(3). Of these, three linear combinations are absorbed by three of the $SU(2)_B \times U(1)_B$ gauge bosons. Four degrees of freedom remain to form the complex Higgs doublet h:

$$H = e^{i\frac{\Sigma}{f}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} \tag{3.7}$$

with

$$\Sigma = \begin{pmatrix} & & & h \\ & O_{3\times 3} & & \\ & & & 0 \\ h^\dagger & 0 & 0 & \end{pmatrix} \qquad (3.8)$$

The relevant mass terms in the top sector result in [5]

$$m_{\text{tA}}^2 \approx \frac{y^2 M^2}{M^2 + y^2 f^2} v^2, \qquad M_{\text{TA}}^2 \approx M^2 + y^2 f^2 - m_{\text{tA}}^2, \qquad (3.9)$$

$$m_{\text{tB}}^2 \approx y^2 f^2, \qquad\qquad M_{TB}^2 \approx M^2. \qquad (3.10)$$

The gauge boson spectrum results in (see appendix B for details)

$$m_\gamma^2 = 0,$$

$$m_{Z_A}^2 = \frac{g^2 f^2}{2 c_W} \sin^2 \frac{v}{f}$$

$$m_{W_A}^2 = \frac{g^2}{2} f^2 \sin^2 \frac{v}{f}, \qquad (3.11)$$

for the A sector, and

$$M_{\gamma_B}^2 = \frac{1}{2} \left[\frac{M_{Z_A}^2}{\tan^2 \frac{v}{f}} + M_B^2 - \sqrt{\left(\frac{M_{Z_A}^2}{\tan^2 \frac{v}{f}} + M_B^2 \right)^2 - 4 M_{W_B}^2 M_B^2} \right], \qquad (3.12)$$

$$M_{Z_B}^2 = \frac{1}{2} \left[\frac{M_{Z_A}^2}{\tan^2 \frac{v}{f}} + M_B^2 + \sqrt{\left(\frac{M_{Z_A}^2}{\tan^2 \frac{v}{f}} + M_B^2 \right)^2 - 4 M_{W_B}^2 M_B^2} \right], \qquad (3.13)$$

$$M_{W_B}^2 = \frac{g^2}{2} f^2 \cos^2 \frac{v}{f}, \qquad (3.14)$$

for the B sector. Expanding in v/f this spectrum coincides with the one obtained in ref. [5].

3.1 The Coleman-Weinberg potential for the twin higgs model in UED

We will now compute the effective potential for the zero-mode Higgs, starting from the contributions from the top sector. The contributions from the zero modes in the top sector result in

$$m_{\text{top}}^2 = \frac{3}{8\pi^2} \frac{y^2 M^2}{M^2 - y^2 f^2} \left[M^2 \ln \frac{M_{\text{TA}}^2}{M^2} - y^2 f^2 \ln \frac{M_{\text{TA}}^2}{m_{\text{tB}}^2} \right], \qquad (3.15)$$

for the term proportional to $h^\dagger h$, whereas for the quartic coupling we obtain

$$\lambda_{\text{top}} = -\frac{m_q^2}{3 f^2} + \frac{3}{16\pi^2} y^4 M^4 \left[\frac{1}{M_{\text{TA}}^4} \ln \frac{M_{\text{TA}}^2}{m_{\text{tA}}^2} + \frac{M_{\text{TA}}^2}{(M^2 - y^2 f^2)^3} \ln \frac{M^2}{m_{\text{tB}}^2} \right]$$

$$- \frac{3}{32\pi^2} y^4 M^4 \left[\frac{1}{M_{\text{TA}}^4} + \frac{4}{(M^2 - y^2 f^2)^2} \right] \qquad (3.16)$$

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in agreement with ref. [5].

We will now consider the contributions from KK modes. Just as in the case of the Little Higgs in section 2, the cutoff of the 5D theory is defined by strong coupling in the KK theory. However, contributions above the Twin Higgs cutoff $\Lambda \simeq 4\pi f$ do not add to the Higgs potential. Thus, the sum over KK modes should be cut at $n_{\mathrm{m}} \simeq \Lambda R$, with Λ the Twin Higgs cutoff rather than the 5D cutoff Λ_{5D} defined in eq. (1.4).

In order to obtain the contributions from the KK fermions we must be careful in keeping all relevant terms in v/f in eq. (B.3). After collecting the quadratic and quartic terms we obtain

$$m^2_{tKK} = \frac{3}{8\pi^2} y^2 M^2 \sum_{n=1}^{n_{\mathrm{m}}} \left\{ \frac{1}{M_{TA}^2} \left(\frac{n^2}{R^2} + M_{TA}^2 \right) \ln \left(1 + \frac{M_{\mathrm{TA}}^2 R^2}{n^2} \right) \right.$$
$$- \frac{1}{M^2 - y^2 f^2} \left(\frac{n^2}{R^2} + M^2 \right) \ln \left(1 + \frac{M^2 R^2}{n^2} \right)$$
$$\left. + \frac{1}{M^2 - y^2 f^2} \left(\frac{n^2}{R^2} + y^2 f^2 \right) \ln \left(1 + \frac{y^2 f^2 R^2}{n^2} \right) \right\}, \quad (3.17)$$

and

$$\lambda_{tKK} = -\frac{m^2_{tKK}}{3f^2} + \frac{3}{16\pi^2} y^4 M^4 \sum_{n=1}^{n_{\mathrm{m}}} \left(\frac{2n^2}{R^2} + M_{\mathrm{TA}}^2 \right) \left\{ \frac{1}{M_{\mathrm{TA}}^6} \ln \left(1 + \frac{M_{\mathrm{TA}}^2 R^2}{n^2} \right) \right.$$
$$\left. + \frac{1}{(M^2 - y^2 f^2)^3} \left[\ln \left(1 + \frac{M^2 R^2}{n^2} \right) - \ln \left(1 + \frac{y^2 f^2 R^2}{n^2} \right) \right] \right\}$$
$$- \frac{3}{8\pi^2} n_{\mathrm{m}} y^4 M^4 \left[\frac{1}{M_{\mathrm{TA}}^4} + \frac{1}{(M^2 - y^2 f^2)^2} \right]. \quad (3.18)$$

As expected from the fact that the Yukawa couplings are SU(4) symmetric, the zero-mode contributions from eqs. (3.15) and (3.16) are finite and regulated by M. This is also the case for the KK contributions in eqs. (3.17) and (3.18). Although one could imagine that summing over the KK modes might re-introduce cutoff sensitivity through the dependence on $n_{\mathrm{m}} \simeq \Lambda R$, this is not the case. This can be seen, for instance, by taking the limit of large n in eqs. (3.17) and (3.18) and seeing that all terms involving n_{m} cancel. The sum over the KK modes does not introduce any dependence on the cutoff Λ in the top sector. We then conclude that the SU(4) symmetric Yukawa couplings are still efficient in regulating the contributions from the top sector KK modes. This will be very important in obtaining suitable solutions for electroweak symmetry breaking with a relatively light Higgs.

The gauge boson contributions are not regulated and therefore do lead to cutoff sensitivity. The zero-mode contributions to the mass parameter are given by

$$m_g^2 = \frac{3}{64\pi^2} \left(3g^2 M_{W_B}^2 \ln \frac{\Lambda^2}{M_{W_B}^2} + g'^2 M_{Z_B}^2 \ln \frac{\Lambda^2}{M_{Z_B}^2} \right), \quad (3.19)$$

in agreement with ref. [5]. The contributions to the quartic coupling are

$$\lambda_g = -\frac{5}{6} \frac{m_{\mathrm{WZ}}^2}{f^2} + \frac{3}{256\pi^2 f^2} \left[3g^2 M_{W_B}^2 \left(1 - 2\ln \frac{\Lambda^2}{M_W^2} \right) + 2g'^2 M_{Z_B}^2 \right] \quad (3.20)$$

The contributions from the gauge boson KK modes read

$$m_{gKK}^2 = \frac{3}{64\pi^2} \left(3g^2 M_{W_B}^2 + g'^2 M_{Z_B}^2\right)(2n_{\mathrm{m}} - \ln 2\pi n_{\mathrm{m}})$$
$$- \frac{3}{64\pi^2} \sum_{n=1}^{n_{\mathrm{m}}} \left\{ 3g^2 \left(\frac{n^2}{R^2} + M_{W_B}^2\right) \ln\left(1 + \frac{M_{W_B}^2 R^2}{n^2}\right) \right.$$
$$\left. + g'^2 \left(\frac{n^2}{R^2} + M_{Z_B}^2\right) \ln\left(1 + \frac{M_{Z_B}^2 R^2}{n^2}\right) \right\}, \qquad (3.21)$$

for the quadratic term in the potential. For the quartic coupling they are given by

$$\lambda_{gKK} = -\frac{m_{gKK}^2}{3f^2} - \frac{3}{64\pi^2 f^2} \left(3g^2 M_{W_B}^2 + g'^2 M_{Z_B}^2\right)(n_{\mathrm{m}} - \ln 2\pi n_{\mathrm{m}}) \qquad (3.22)$$
$$+ \frac{3}{128\pi^2 f^2} \sum_{n=1}^{n_{\mathrm{m}}} \left\{ 3g^2 M_{W_B}^2 \ln\left(1 + \frac{M_{W_B}^2 R^2}{n^2}\right) + \frac{1}{2}g'^4 f^2 \ln\left(1 + \frac{M_{Z_B}^2 R^2}{n^2}\right) \right\}$$

Unlike the contributions from the top sector KK modes, the gauge boson KK modes do result in linear divergences. This appears as a linear dependence on $n_{\mathrm{m}} \simeq \Lambda R$. However, this will not result in a heavy Higgs, since the gauge boson contributions tend to lower the Higgs mass.

3.2 Results and discussion

We combine all the contributions to the Coleman-Weinberg potential obtained in the previous section to look for solutions with electroweak symmetry breaking and a light Higgs. We consider two cases: $f = 1/R$ and $f = 3/R$.

In the first case, with $f = 1/R$, the cutoff of the 5D theory is well above the Twin Higgs model cutoff of $\Lambda_{\mathrm{TH}} \simeq 4\pi f$. Then, just as in the Little Higgs model of section 2, we only sum KK mode contributions up to $n_{\mathrm{m}} \simeq \Lambda_{\mathrm{TH}} R = 4\pi$.

| M_B | M | f | m_h |
(TeV)	(TeV)	(GeV)	(GeV)
1	1.5	850	230
1	3	2000	560
2	2	1200	270
3	1.5	1300	280
5	2	850	170

Table 2: Solutions for $f = 1/R$ for the Twin Higgs Model in one UED. The number of KK modes are summed up to $n_{\mathrm{m}} = 4\pi$.

In table 2 we show some representative solutions for this case in which the scale f is adequately large and the Higgs mass not too heavy. Solutions with larger values of M would tend to have larger values of m_h since M regulates the contributions from the top sector to the Higgs mass. On the other hand, solutions with larger values of M_B result in a smaller m_h, since M_B enhances the gauge boson contributions, which make the Higgs lighter. The additional fields in the Twin Higgs model have no or very little couplings with the SM fields, with the main exception being the Higgs. Thus, the phenomenology of this scenario is very similar to the one for the SM in one UED. The exception is the presence of a heavy singlet quark plus an $\mathrm{SU}(2)_L$ doublet, colorless twin-quark. The zero modes of these states appear only at the scale M, which could be not too far above the compactification scale R^{-1}. The twin gauge bosons

are, in principle decoupled from SM fields. The exotic quarks $\tilde{q}_A$ and $\tilde{q}_B$ induce kinetic mixing between the SM photon and the twin photon. However, this does not affect collider phenomenology in any significant way.

The motivation for the second case we study is to find a scenario where f and $1/R$ are sufficiently separated so as to make it possible to have an energy interval above the current experimental limits of $1/R > 300\,\mathrm{GeV}$, where the theory is just the SM in one UED, without any additional states and with a rather low $1/R$. As an example we consider $f = 3/R$. In table 3 we show representative examples in this scenario.

| M_B | M | f | m_h |
(TeV)	(TeV)	(GeV)	(GeV)
1	2	1300	500
2	1	950	300
3	2	1600	320
4	2	1700	270

Table 3: Solutions for $f = 3/R$ for the Twin Higgs Model in one UED. The number of KK modes are summed up to $n_{\mathrm{m}} = 12\pi$.

In this case, the sum over the KK modes must be extended up to $n_{\mathrm{m}} = \Lambda_{\mathrm{TH}} R \simeq 12\pi$. As before, the lighter Higgs masses are obtained for larger values of M_B. For instance, the last entry in table 3 is $M_B = 4$ TeV, $M = 2$ TeV and $f = 1.7$ TeV, which corresponds to $R^{-1} \simeq 570$ GeV. This is in agreement with current experimental bounds, both on R^{-1} as well as on the new states associated with the scale f. Then in these kind of scenarios, it is possible to stabilize the Higgs mass in what it would appear — at the Tevatron and perhaps even at the early LHC– as the SM in one universal extra dimension. However, the LHC should eventually discover the extended fermion sector. Their zero modes would have masses of order M, and their KK modes would start not far above, typically at $M^2 + 1/R^2$. Thus, the discovery at the LHC of the Twin Higgs states would always be accompanied by the discovery of a few of the corresponding KK modes.

We finally note that, unlike in the 4D case, a μ term was not necessary in order to obtain acceptably light Higgs masses. The use of a μ term would result in even smaller values of m_h relative to what is shown in table 3.

4. Conclusions

Theories with universal extra dimensions, in which the entire SM field content propagates in the 5D bulk, suffer from a little hierarchy, as discussed in section 1. We have seen that it is possible to control the Higgs mass if the Higgs is a pseudo-Goldstone boson living in the 5D bulk. This is so despite of the fact that the divergences in these 5D theories are linear in the cutoff, whereas in 4D they are logarithmic. We considered two scenarios to solve this little hierarchy problem: a Little Higgs model and a Twin Higgs model.

As an example of the Little Higgs case we studied the Simplest Little Higgs model of ref. [4] in one universal extra dimension. The cutoff dependence, which is logarithmic in four-dimensional Little Higgs models, is linear in 5D models and tends to make the Higgs heavier. Despite this difficulty, we have found solutions with relatively low Higgs masses $m_h \simeq (250 - 260)$ GeV, for values of the symmetry breaking scale f that are high enough to satisfy direct as well as indirect constraints. These solutions though, correspond to the case in which $f \sim 1/R$. Then, the phenomenology of this solution to the Little Hierarchy

in UED theories is markedly different than the one corresponding to the SM in UED [12], since the appearance of the first KK modes of the SM fields is always accompanied by the zero-modes of the new states in the Little Higgs spectrum, such as the new gauge bosons and fermions.

We also considered a Twin Higgs model in one universal extra dimension. Just as in the Little Higgs case, we are able to stabilize m_h despite the linear divergences. We do so without the need of a μ term, which was necessary in the Little Higgs case, as well as in the Twin Higgs in 4D. Furthermore, in the Twin Higgs case we are also able to find solutions where the scales f and $1/R$ are separated, in addition to solutions with $f \sim 1/R$. Thus, the use of the Twin Higgs mechanism allows the UED scenario with only the SM fields to be realized, with $1/R$ not far above the weak scale and f at or somewhat above the TeV scale, as it can be seen in table 3. In both Twin Higgs scenarios, $f \sim 1/R$ and $f > 1/R$, the phenomenology is very similar to that of the SM in UED, due to the fact that the new Twin states appearing at the scale f and above tend to have no or very little interactions with the SM fields. The exception is the presence of heavy quarks at the scale M. Then, generically, the Twin Higgs mechanism stabilizes the Higgs mass in the extra dimensional theory without major changes in the phenomenology. This is particularly true for $f \simeq 1/R$, since in this case M –and with it the new heavy quark state– are likely to be beyond the reach of the LHC.

We then conclude that if the KK modes corresponding to the SM in UED are observed without additional states, the mechanism for stabilizing the Higgs mass is probably the Twin Higgs, but certainly not a Little Higgs.

We notice that 6D UED theories, of great interest due to a variety of theoretical [14] and phenomenological [15] issues, have a stronger cutoff dependence. From an argument analogous to the one leading to eq. (2.35), we see that the 6D embedding of a theory with the required global symmetries to protect the Higgs mass, would result in a quadratic cutoff dependence. This will always be the case as long as the explicit symmetry breaking leads to a logarithmic cutoff dependence in the corresponding 4D theory. Repeating the argument of eq. (2.35) for the simplest Little Higgs case, now the one loop contribution to $|\Phi_1^\dagger \Phi_2|^2$ results, in the 6D UED theory, in a contribution to the Higgs mass of the form

$$\frac{g^4 f^2}{16\pi} \, (\Lambda R)^2 \, , \tag{4.1}$$

which depends quadratically on the cutoff of the Little Higgs theory. In the KK picture, this will appear as a quadratic dependence on the the maximum number of KK modes to be summed. Then in principle, stabilizing the Higgs mass in a 6D theory appears to be a rather difficult task. It would probably be necessary to have an additional global symmetry in the top sector, such as in the Twin Higgs model presented in section 3, in order to protect the Higgs mass from the large contributions of this sector, which if quadratic in the cutoff would probably make the Higgs unacceptably heavy.

Acknowledgments

A. The mass spectrum in the simplest little Higgs model

In this appendix we derive the expressions for the mass matrices both of fermions and gauge bosons used in section 2.1.

In order to obtain the mass matrices, we expand the fields in eq. (2.1) taking $\langle h \rangle = (0 \; v)^T$. Expanding the exponentials in the VEVs results in

$$\langle \Phi_1 \rangle = f_1 \begin{pmatrix} is_1 \\ 0 \\ c_1 \end{pmatrix} \qquad \langle \Phi_2 \rangle = f_2 \begin{pmatrix} -is_2 \\ 0 \\ c_2 \end{pmatrix}, \tag{A.1}$$

where we have defined

$$s_1 \equiv \sin\left(\frac{f_2 v}{f_1 f}\right) \qquad c_1 \equiv \cos\left(\frac{f_2 v}{f_1 f}\right)$$

$$s_2 \equiv \sin\left(\frac{f_1 v}{f_2 f}\right) \qquad c_2 \equiv \cos\left(\frac{f_1 v}{f_2 f}\right). \tag{A.2}$$

The mass eigenvalues for a given particle type can be written as

$$m_\pm^2 = \frac{1}{2}\left[M(f)^2 \pm \sqrt{M(f)^4 - 4\delta(f,v)} \right]$$

$$\approx \frac{1}{2}\left[M(f)^2 \pm M(f)^2 \right] \mp \frac{\delta(f,v)}{M(f)^2}\left[1 + \frac{\delta(f,v)}{M(f)^4} \right]. \tag{A.3}$$

In eq. (A.3), $M(f)^2$ is the dominant part of the heavy partner mass, mostly associated with the eigenvalue m_+^2, and all dependence of the Higgs VEV v appears only in $\delta(f,v)$. In what follows we will compute $M(f)^2$ and $\delta(f,v)$ for fermions and gauge bosons.

We first consider the fermion spectrum, and in particular the top and its partner as it is the dominant fermion contribution. Their quadratic masses are obtained squaring the mass matrix of the bi-linears in t and T

$$\mathcal{L}_{t,T} = \overline{\Psi}_{Q_3} \left(\lambda_{t1}\langle\Phi_1\rangle \; \lambda_{t2}\langle\Phi_2\rangle \right) \begin{pmatrix} t_R \\ T_R \end{pmatrix} + h.c.$$

$$= \overline{\Psi}_{Q_3} \mathcal{M}_{Q_3}^\dagger \begin{pmatrix} t_R \\ T_R \end{pmatrix} + h.c., \tag{A.4}$$

which gives

$$\mathcal{M}_{Q_3} \mathcal{M}_{Q_3}^\dagger = \begin{pmatrix} \lambda_{t1}^2 \langle\Phi_1\rangle^\dagger \langle\Phi_1\rangle & \lambda_{t1}\lambda_{t2}\langle\Phi_1\rangle^\dagger\langle\Phi_2\rangle \\ \lambda_{t1}\lambda_{t2}\langle\Phi_2\rangle^\dagger\langle\Phi_1\rangle & \lambda_{t2}^2\langle\Phi_2\rangle^\dagger\langle\Phi_2\rangle \end{pmatrix}. \tag{A.5}$$

This results in

$$\delta_t = \lambda_{t1}^2 \lambda_{t2}^2 f_1^2 f_2^2 \sin^2\left(\frac{fv}{f_1 f_2}\right) \tag{A.6}$$

$$M_T^2 = \lambda_{t1}^2 f_1^2 + \lambda_{t2}^2 f_2^2. \tag{A.7}$$

Then, up to order v^4/f^4, the mass squared of the top quark and its partner are

$$m_t^2 = (\lambda_{t1}\lambda_{t2})^2 \frac{f^2 v^2}{M_T^2} \left[1 - \frac{1}{3}\left(\frac{fv}{f_1 f_2}\right)^2 + (\lambda_{t1}\lambda_{t2})^2 \left(\frac{fv}{M_T^2}\right)^2 \right] \tag{A.8}$$

$$m_T^2 = M_T^2 - m_t^2. \tag{A.9}$$

We now consider the zero-mode gauge boson spectrum of the model. From the nine gauge bosons of the $SU(3)_w \times U(1)_X$ sector eight linear combinations will absorb eight of the twelve degrees of freedom in the scalar triplets. The bi-linears of the gauge bosons come from the following part of the Lagrangian

$$\mathcal{L}_{g.b} = \sum_{i=1,2} \left| \left(g W_\mu^a T^a - \frac{1}{3} g_X B_\mu \right) \langle \Phi_i \rangle \right|^2 \tag{A.10}$$

Among the neutral gauge bosons there will be two massive states, Z_1 and Z_2, resulting from combinations of W_3, W_8 and B, two states from combinations of W_5 and W_5 that we call U^0, and the photon.

$$\mathcal{L}_{neut.}^{mass} = \left(W_3 \ W_8 \ B \ W_5 \right) \frac{\mathcal{M}_{neu}^2}{2} \begin{pmatrix} W_3 \\ W_8 \\ B \\ W_5 \end{pmatrix} + \frac{g^2}{4} f^2 W_4^\mu W_{4\mu} \tag{A.11}$$

After electroweak symmetry breaking, the neutral gauge boson squared mass matrix is given by

$$\mathcal{M}_{neu}^2 = \frac{g^2}{2} \begin{pmatrix} a & \frac{1}{\sqrt{3}}a & -\frac{2b}{3}a & -u \\ \frac{1}{\sqrt{3}}a & -a + \frac{4}{3}f^2 & -\frac{2b}{3\sqrt{3}}(3a - 2f^2) & \frac{1}{\sqrt{3}}u \\ -\frac{2b}{3}a & -\frac{2b}{3\sqrt{3}}(3a - 2f^2) & \frac{4b^2}{9}f^2 & \frac{4b}{3}u \\ -u & \frac{1}{\sqrt{3}}u & \frac{4b}{3}u & f^2 \end{pmatrix} \tag{A.12}$$

with the definitions

$$a = f_1^2 s_1^2 + f_2^2 s_2^2 \tag{A.13}$$

$$b^2 = \frac{3t^2}{3 - t^2} \tag{A.14}$$

$$u = f_1^2 s_1 c_1 - f_2^2 s_2 c_2. \tag{A.15}$$

Diagonalization of eq. (A.12) results in

$$\delta_z = g^4 \frac{(1 + t^2)}{3 - t^2} f_1^2 f_2^2 (s_1 c_2 + s_2 c_1)^2 \tag{A.16}$$

$$M_{Z'}^2 = \frac{2}{3 - t^2} g^2 f^2, \tag{A.17}$$

with $t = g'/g = \tan(\theta_W)$.

Therefore, up to order v^4/f^4, the masses of Z_1, Z_2 and U^0 are

$$M_{Z_1}^2 = m_{Z^0}^2 \left[1 - \frac{1}{3} \frac{v^2 f^2}{f_1^2 f_2^2} + \frac{m_{Z^0}^2}{M_{Z'}^2} \right] \qquad \text{(A.18)}$$

$$M_{Z_2}^2 = M_{Z'}^2 - M_{Z_1}^2 \qquad \text{(A.19)}$$

$$M_{U^0}^2 = \frac{1}{2} g^2 f^2 \, , \qquad \text{(A.20)}$$

where $m_Z^2 = g^2 v^2 / 2 \cos^2 \theta_W$ is the tree level squared mass of the SM Z.

For the charged gauge bosons we have four states which are linear combinations of the symmetry eigenstates as $W_I^\pm = 1/\sqrt{2}(W_1 \mp iW_2)$ and $W_{II}^\pm = 1/\sqrt{2}(W_6 \pm iW_7)$. Schematically we have

$$\mathcal{L}_{\text{ch}}^{\text{mass}} = \left(W_I^+ \ W_{II}^+ \right) \mathcal{M}_W^2 \begin{pmatrix} W_I^- \\ W_{II}^- \end{pmatrix} \qquad \text{(A.21)}$$

with the squared mass matrix given by

$$\mathcal{M}_W^2 = \frac{g^2}{2} \begin{pmatrix} a & -iu \\ iu & f^2 - a \end{pmatrix} \qquad \text{(A.22)}$$

This results in the parameters of the charged sector spectrum

$$\delta_W = \frac{g^4}{4} f_1^2 f_2^2 \left(s_1 c_2 + s_2 c_1 \right)^2 \qquad \text{(A.23)}$$

$$M_{\text{Ch}}^2 = \frac{1}{2} g^2 f^2 , \ . \qquad \text{(A.24)}$$

Then, up to order v^4/f^4 the mass squared of the diagonal mass eigenstates $W^\pm$ and $W'^\pm$ which are mixing of $W_I^\pm$ and $W_{II}^\pm$ are

$$M_W^2 = m_W^2 \left[1 - \frac{1}{3} \frac{v^2 f^2}{f_1^2 f_2^2} + \frac{m_W^2}{M_{\text{Ch}}^2} \right] \qquad \text{(A.25)}$$

$$M_{W'}^2 = M_{\text{Ch}}^2 - M_W^2 \qquad \text{(A.26)}$$

where $m_W^2 = g^2 v^2 / 2$ is the squared mass of the SM W at tree level.

B. The mass spectrum in the twin Higgs model

Here we show in some detail the calculation of the mass matrices for fermions and gauge bosons, used in the text to compute the Coleman-Weinberg potential in the Twin Higgs model. The VEV of the scalar doublet, $\langle h \rangle = v$, leads to

$$\langle H \rangle = \begin{pmatrix} if \sin \frac{v}{f} \\ 0 \\ 0 \\ f \cos \frac{v}{f} \end{pmatrix} = \begin{pmatrix} \langle H \rangle_A \\ \langle H \rangle_B \end{pmatrix} \qquad \text{(B.1)}$$

With this VEV structure we can also find the mass eigenvalues in a closed form, which will be particularly useful in computing the Coleman-Weinberg potential in the 5D UED theory. This non-linear description is valid up to the cutoff $\Lambda \simeq 4\pi f$.

The mass matrix in the quark sector is

$$\mathcal{L}_q^{\text{mass}} = \left(t_A \ t_B \ \tilde{q}_A^c \ \tilde{q}_B^c \right) \begin{pmatrix} y\langle H\rangle_A^\dagger & 0 & y\langle H\rangle_B^\dagger & 0 \\ 0 & y\langle H\rangle_B^\dagger & 0 & y\langle H\rangle_A^\dagger \\ 0 & 0 & M & 0 \\ 0 & 0 & 0 & M \end{pmatrix} \begin{pmatrix} q_A \\ q_B \\ \tilde{q}_A \\ \tilde{q}_B \end{pmatrix} + h.c \quad (B.2)$$

which results in the eigenvalues

$$\begin{aligned} m_{\text{tA}}^2 &= \frac{1}{2}\left[M^2 + y^2 f^2 - \sqrt{(M^2 + y^2 f^2)^2 - 4\delta_1} \right] \\ m_{\text{TA}}^2 &= \frac{1}{2}\left[M^2 + y^2 f^2 + \sqrt{(M^2 + y^2 f^2)^2 - 4\delta_1} \right] \\ m_{\text{tB}}^2 &= \frac{1}{2}\left[M^2 + y^2 f^2 - \sqrt{(M^2 + y^2 f^2)^2 - 4\delta_2} \right] \\ m_{\text{TB}}^2 &= \frac{1}{2}\left[M^2 + y^2 f^2 + \sqrt{(M^2 + y^2 f^2)^2 - 4\delta_2} \right], \end{aligned} \quad (B.3)$$

with

$$\delta_1 = y^2 f^2 M^2 \sin^2 \frac{v}{f} \quad (B.4)$$

$$\delta_2 = y^2 f^2 M^2 \cos^2 \frac{v}{f} \quad (B.5)$$

The gauge boson mass matrix is obtained from the interactions with the pNGBs

$$\mathcal{L}_{g.b} = \sum_{j=A,B} \left| \left(g W_{j\mu}^a \tau_j^a - \frac{g'}{2} B_{j\mu} \right) \langle H_j \rangle \right|^2 \quad (B.6)$$

where τ_j^a are the usual SU(2) generators. We also consider an explicit mass term M_B for the abelian gauge field $B_{B\mu}$, which results in a mass for the twin photon and breaks the twin symmetry softly. This mass could be generated dynamically at scales above the cutoff Λ. The mass eigenvalues for the A sector can be written as

$$\begin{aligned} m_\gamma^2 &= 0, \\ m_{Z_A}^2 &= \frac{g^2 f^2}{2c_W} \sin^2 \frac{v}{f} \\ m_{W_A}^2 &= \frac{g^2}{2} f^2 \sin^2 \frac{v}{f} \end{aligned} \quad (B.7)$$

and for the sector B

$$M_{\gamma_B}^2 = \frac{1}{2}\left[\frac{M_{Z_A}^2}{\tan^2\frac{v}{f}} + M_B^2 - \sqrt{\left(\frac{M_{Z_A}^2}{\tan^2\frac{v}{f}} + M_B^2\right)^2 - 4M_{W_B}^2 M_B^2}\,\right], \qquad (\text{B.8})$$

$$M_{Z_B}^2 = \frac{1}{2}\left[\frac{M_{Z_A}^2}{\tan^2\frac{v}{f}} + M_B^2 + \sqrt{\left(\frac{M_{Z_A}^2}{\tan^2\frac{v}{f}} + M_B^2\right)^2 - 4M_{W_B}^2 M_B^2}\,\right], \qquad (\text{B.9})$$

$$M_{W_B}^2 = \frac{g^2}{2}f^2\cos^2\frac{v}{f}\ . \qquad (\text{B.10})$$

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